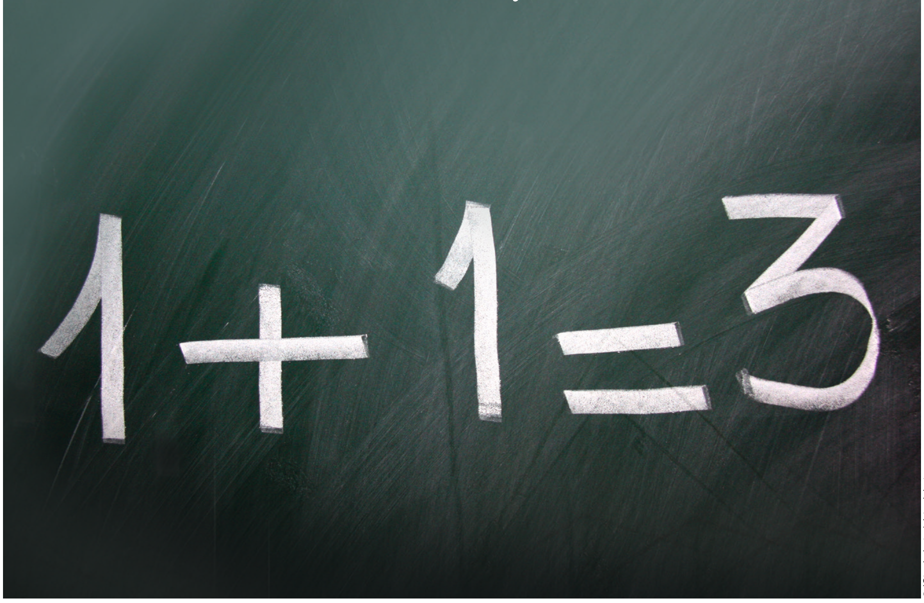
Math on trial (Pt 2)

Dr Chris Pamplin looks at some common mathematical errors that have led courts astray, and how to avoid them



The first part of this short series looked at *Math on Trial* (Schneps, L & Colmez, C, 2013, Basic Books), an excellent book that catalogues the use—or perhaps that should be misuse—of mathematics in the courtroom (see "Math on trial (Pt 1)", NLJ, 5 June 2015, p 19). While the publication is well worth reading in its entirety, the purpose here is to summarise the ten common mathematical errors the authors distil from the legal casebook. Last time we looked at:

(1) Multiplying non-independent probabilities.
(2) Making unjustified estimates.
(3) Getting something from nothing.
(4) The value in re-running experiments.
(5) The birthday problem.

As the authors say, "despite their ubiquity… most of these fallacies are easy to spot". This two-part series offers your very own fallacy-spotting crib sheet.

Error no 6: Simson's paradox

Simson's Paradox arises when a trend disappears (or reverses) when the groups showing the trend are combined. The classic legal case demonstrating the point is the University of California, Berkeley sex discrimination case. The two groups (male and female applicants to Berkeley) show a clear bias in favour of males. But when considering all applicants to given departments across Berkeley, the male bias vanishes.

The Berkeley admission figures for the autumn of 1973 showed that 8,442 men applied and 44% were admitted, whereas only 35% of the 4,321 women who applied were admitted. This difference was so large that it was unlikely to be down to chance. However, when you consider the individual departments, no department was significantly biased against women. In fact, most departments had a small but statistically significant bias in favour of women.

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The resolution of this paradox is that women tended to apply to competitive departments with low rates of admission even among qualified applicants (such as the English department), whereas men tended to apply to less-competitive departments with high rates of admission among the qualified applicants (such as in engineering and chemistry).

Simson's Paradox tells us that it can be easy to "cut the data" to prove a particular point, but doing so will involve hiding some important factor or other. Look carefully!

Error no 7: incredible coincidence

The conviction of Lucia de Berk as a serial killer of children is an example of the error of analysing data with a preconceived idea of what that data will tell you.

In 2003 Lucia de Berk was sentenced to life imprisonment in the Netherlands for four murders and three attempted murders of patients in her care. In 2004, after an appeal, she was convicted of seven murders and three attempted murders. Her conviction was controversial, and in 2008 the case was reopened by the Supreme Court of the Netherlands. She was freed, her case was re-tried and she was exonerated in April 2010.

An inexpert statistical analysis was used to proclaim that the chance of a nurse working at the three hospitals involved being present at the scene of so many unexplained deaths and resuscitations was 1 in 342 million. However, the evidence gathering had been undertaken by hospital administrators once they suspected de Berk of being the killer.

Events were attributed to de Berk once suspicions began to fall on her that in reality could not have had anything to do with her. Once the failings in the source data were corrected, it was calculated that there was a chance of 1 in 25 that a nurse could experience a sequence of events of the same type as Lucia de Berk.

The lesson here is that retrospective thinking, and particularly attempting to retroactively determine probabilities for events that have already happened, is a very slippery slope.

Error no 8: underestimation

As humans we are used to dealing with small numbers, but very large numbers tend to confound us. We think on a human scale and appear to have difficulty using our "common sense" to make intuitive predictions when large numbers are involved.

A fine example of this is the "girdled Earth". Without doing the maths, imagine a cable laid on the ground that runs around the entire equator of the Earth. Let's assume that it is 40,000km long. Now, imagine making the cable 1 metre longer. This longer cable will be raised a bit off the ground because it is a little bit longer, but how far off the ground does your intuition tell you it will be? Would you be able to get a sheet of paper under it?

The answer is 16 cm. That answer, to many, is an astonishing result, but the maths is simple and the answer indisputable.

This type of mathematical error enters the courtroom in fraud cases, like that of Charles Ponzi or Bernie Madoff, who promote schemes that can only work if they deliver exponential growth on investments. Such schemes cannot work. The lesson to take away is that human intuition, which is fragile at the best of times, is particularly weak when large numbers or compound growth is involved.

Error no 9: choosing the wrong model

Our penultimate error is provided by a battle over the will of Sylvia Howland. In an attempt to demonstrate that a signature on the second page of the will had been forged, it being thought too similar to the signature on the first page, the prosecution turned to a Harvard professor of mathematics.

He compared 42 examples of Sylvia Howland's signature, giving 861 individual comparisons. From the number of down strokes that coincided on each comparison, he calculated that the chance of her producing two such similar signatures was vanishingly small.

But his model was too simplistic. It took no account of the possibility of her signature changing gradually over time, so two signatures made close together may be more similar than two signatures made years apart. Furthermore, it failed to account for the possibility that signatures made with the same pen at the same table in quick succession might be more similar than two signatures made in different settings.

Mathematical models always simplify the real world, and the simpler the model, the more danger there is of the model turning out nonsense. When such models end up in court, there is a real danger of injustice.

Error no 10: the likelihood of unlikely events

The Dreyfus Affair was one of the most famous trials of the 19th century, but his conviction was based upon a misunderstanding that the likelihood of unlikely events is dependent on how many attempts are made. So this is error 7 applied to a set of unlikely events.

The scandal began in December 1894, with the treason conviction of Captain Alfred Dreyfus, a young French artillery officer of Alsatian and Jewish descent. Sentenced to life imprisonment for allegedly communicating in a letter French military secrets to the German Embassy in Paris, Dreyfus was imprisoned on Devil's Island in French Guiana. In 1906 Dreyfus was exonerated and reinstated as a major in the French Army.

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A key piece of evidence used against Dreyfus was that the repeated placement of certain words in the letter with respect to feint lines in the paper was too unlikely to be coincidence, and so must reveal careful planning by the author to convey some hidden meaning. The "expert" who conducted this piece of work was convinced there was a 1 in 400,000 chance of the pattern he saw being due to chance. But by failing to recognise that in focusing on the words he selected, he missed the placement of all the other words in the letter, his calculated chance was extremely wide of the mark. In fact, there was a 13 in 100 chance of what he saw in the letter happening by chance.

To explain this another way, should you be surprised if you see an archer get eight arrows in the bull's-eye? The answer lies in how many arrows he fired in total. If he fired 10 arrows and eight hit the mark that is unusual. If the area around the target is littered with hundreds of fallen arrows, the feat is less surprising.

The Dreyfus Affair shows us that uncommon things will occur if you try often enough. It's why the lottery works —any individual ticket holder has vanishingly small odds of winning, but with millions of tickets sold each week it is unsurprising that week after week somebody does win with those vanishingly small odds.

Conclusion

The ten errors covered by the authors of *Math on Trial* are not the end of the story. There are other mathematical conundrums with which to contend. For example, in a recent Radio 4 discussion on the effect of free schools, we were told that, on average, when a free school opens it makes no difference to the educational standards in the local area. But, enthused one contributor, when you look just at those free schools that opened in areas where the educational standard was low, the standards in all the local schools rose. "Ahh…" came back the other contributor, "… but when a free school opens in an area with schools that are performing very well, those schools tend to get worse". What does this tell us about the impact of free schools? Probably very little. It should simply remind us of the principle of *regression to the mean*: on average, under-performing schools will tend to get better and over-performing schools will tend to get worse.

When mathematics is misused, whether by mistake or design, be it by politicians, the media or commercial operators, the audience will be confounded. If such misuse does little more than annoy us, it perhaps doesn't matter too much. But when these mathematical errors appear in trials, they can, at the extreme, be the difference between life and death.

When you are faced with mathematics in your expert witness practice, bear in mind these 10 common errors. Of course, this short article can't do the errors full justice. For that you should get hold of a copy of *Math on Trial* and read it through. **NLJ**

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